Pearson

Topic 11

Vectors

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

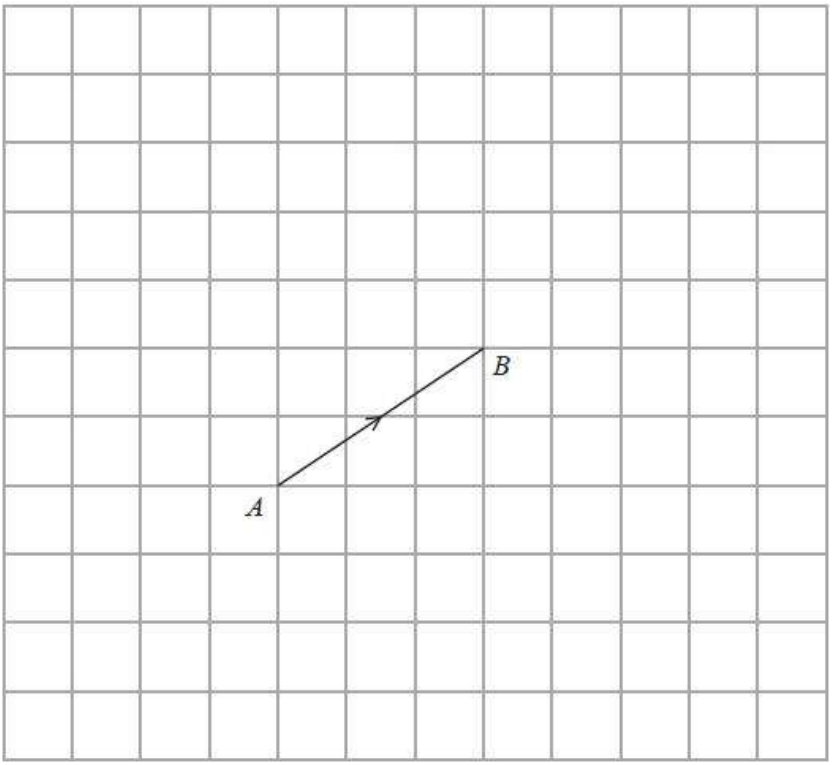
Calculators may not be used

The total mark for this section is 27

1

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

$\overrightarrow{AB}$ is shown on the grid



(a) On the grid, draw $\overrightarrow{BC}$.

(1)

$\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{BC}$

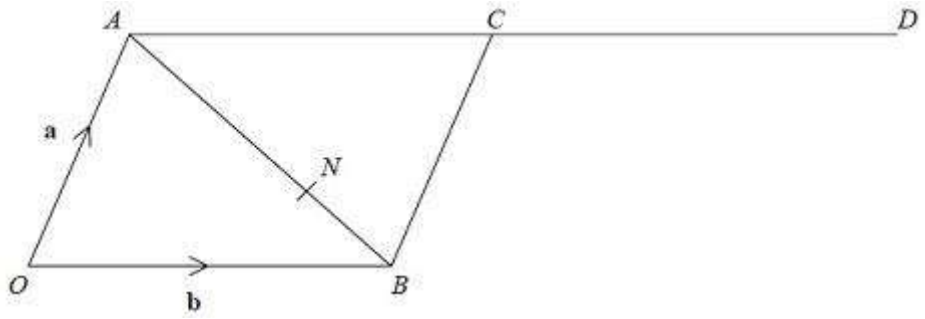
(b) On the grid, mark with a cross (X) the position of D.

Label this point D.

(2)

(Total for Question 1 is 3 marks)

Q2



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$

The point N divides AB in the ratio 2:1

(a) Write an expression for $\overrightarrow{ON}$ in terms of **a** and **b**.

(3)

(b) Prove that OND is a straight line.

(3)

(Total for Question 2 is 6 marks)

Q3

Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q4

A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(Total for Question 4 is 5 marks)

Q5

[In this question, the unit vectors **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.]*

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P, giving your answer as a simplified surd.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

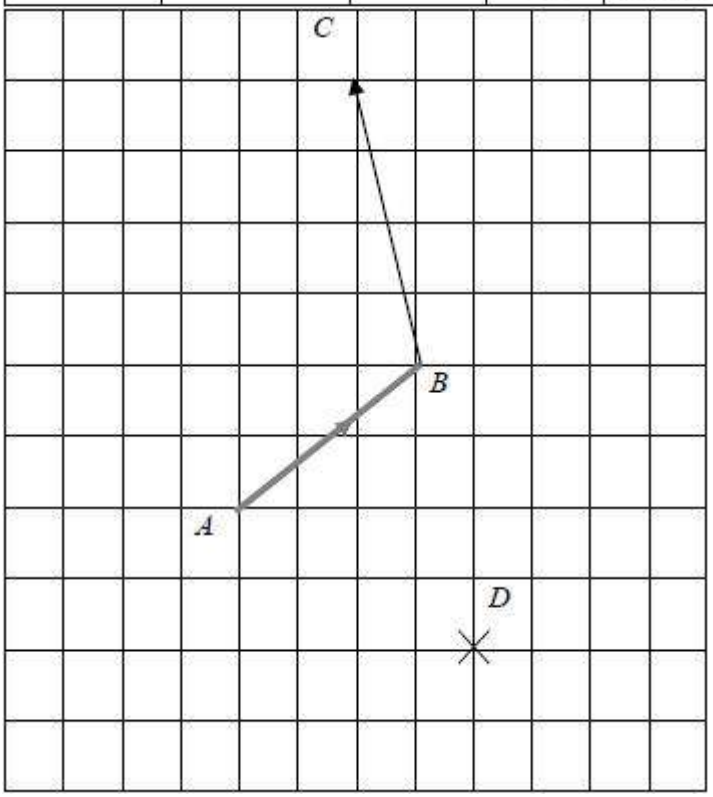
(Total for Question 5 is 9 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Working	Answer	Mark	Notes
(a)		Vector drawn	B1	for correct vector
(b)		× marked	M1	for method to find the vector $\overrightarrow{AD} = \begin{pmatrix} 3--1 \\ 2-4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ or for $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ drawn on the grid
			A1	for ft for correct position (×) D on their diagram



Q2

Question		Working	Answer	Mark	Notes
	(a)	$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{ON} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$ $\overrightarrow{ON} = \mathbf{a} + \frac{2}{3}(-\mathbf{a} + \mathbf{b})$ $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ OR $\overrightarrow{ON} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA}$ $\overrightarrow{ON} = \mathbf{b} + \frac{1}{3}(-\mathbf{b} + \mathbf{a})$ $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	$\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	3	M1 for correct vector equation involving $\overrightarrow{ON}$, eg. $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$, may be written, partially or fully, in terms of **a** and **b**, e.g. $(\overrightarrow{ON} =)\ \mathbf{a} + \frac{2}{3}\overrightarrow{AB}$ M1 for showing answer requires $\overrightarrow{AN} = \frac{2}{3}\overrightarrow{AB}$ or $\overrightarrow{BN} = \frac{1}{3}\overrightarrow{BA}$ A1 $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ oe
	(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CD}$ $= \mathbf{a} + \mathbf{b} + \mathbf{b}$ $= \mathbf{a} + 2\mathbf{b}$ $\overrightarrow{OD} = 3(\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b})$ $\overrightarrow{OD} = 3\overrightarrow{ON}$	Proof	3	M1 for a correct vector statement for $\overrightarrow{OD}$ or $\overrightarrow{ND}$ in terms of **a** and **b**, e.g. $\overrightarrow{OD} = \mathbf{a} + \mathbf{b} + \mathbf{b}$ oe or $\overrightarrow{ND} = \frac{2}{3}(-\mathbf{b} + \mathbf{a}) + \mathbf{b} + \mathbf{b}$ oe A1 for correct and fully simplified vectors for $\overrightarrow{ON}$ (may be seen in (a)) **and** for $\overrightarrow{OD}$ (= **a** + 2**b**) or $\overrightarrow{ND}$ $(= \frac{2}{3}\mathbf{a} + \frac{4}{3}\mathbf{b})$ C1 (dep on A1) for statement that $\overrightarrow{OD}$ or $\overrightarrow{ND}$ is a multiple of $\overrightarrow{ON}$ (+ common point)

Q3.

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\lvert AB \rvert = 3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$

There must be some attempt to write in vector form.

A1: cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\mathbf{i} + 3\mathbf{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\mathbf{i} \\ 3\mathbf{j} \end{pmatrix}$

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $\lvert AB \rvert = \sqrt{(9)^2 + (3)^2}$ is also correct.

Condone missing brackets in the expression $\lvert AB \rvert = \sqrt{-9^2 + (3)^2}$

Also allow a restart usually accompanied by a diagram.

A1ft: $\lvert AB \rvert = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question

Q4.

Question Number	Scheme	Marks
	$(-4\mathbf{i}-7\mathbf{j})=\mathbf{r}+4(-3\mathbf{i}+2\mathbf{j})$ $\mathbf{r}=(8\mathbf{i}-15\mathbf{j})$ $\lvert\mathbf{r}\rvert=\sqrt{8^2+(-15)^2}=17$ m	M1 A1 A1 M1 A1 ft [5]

Q5.

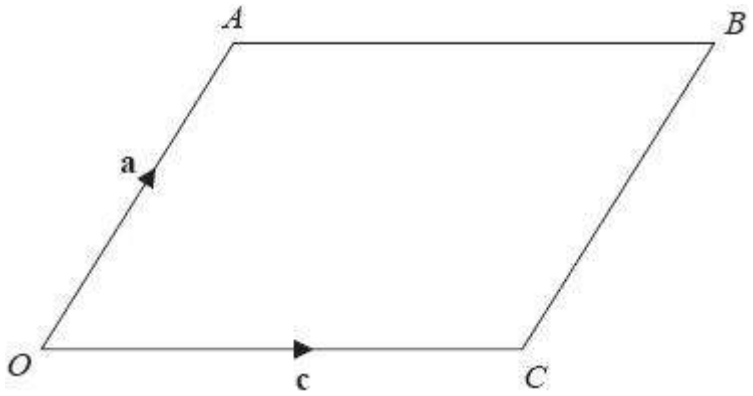
Question Number	Scheme	Marks
(a)	$\sqrt{\left((-4)^2+8^2\right)}=\sqrt{80}$ $\left(\text{km h}^{-1}\right)$ accept exact equivalents or 8.9 or better	M1 A1 (2)
(b)	$\mathbf{p}=(2\mathbf{i}-8\mathbf{j})+t(-4\mathbf{i}+8\mathbf{j})$	B1 (1)
(c)	Equating **j** components $-8+8t=12-8t$ $t=\frac{5}{4}$ oe	M1 A1 A1 (3)
(d)	Using their t from (c) to find the **i**-cpts of **p** and **q** and subtract them $10\frac{1}{2}-(-3)=13\frac{1}{2}$ (km)	M1 A1 ft A1 (3) **9**

Silver Questions

Calculators may not be used

The total mark for this section is 25

Q1



$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of k.

(Total for Question 1 is 4 marks)

Q2

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $\left|\overrightarrow{AB}\right|$. Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q3

Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q.

(Total for Question 3 is 6 marks)

Q4

*[In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.]*

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

(i) parallel to $\mathbf{j}$,
(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

(Total for Question 4 is 11 marks)

End of Questions

Silver Mark Scheme

Q1.

Question	Working	Answer	Mark	Notes
		$\frac{2}{5}$	P1	for first step to solve the problem e.g. $\overrightarrow{AC} = -\mathbf{a} + \mathbf{c}$ or $\overrightarrow{OX} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$ or demonstrates the location of D and X on the diagram
			P1	for a correct vector statement using $\overrightarrow{CD}$ eg $\overrightarrow{CD} = \overrightarrow{CX} + \overrightarrow{XD}$ or $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ or $\overrightarrow{OD} = \frac{7}{2}\mathbf{c}$ or $\overrightarrow{CD} = 2.5\mathbf{c}$ oe
			P1	for a correct equation or ratio using k eg equating $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a} = \frac{1}{2}(-\mathbf{a} + \mathbf{c}) + \frac{1}{k}\mathbf{c}$ or $\frac{\overrightarrow{OD}}{\overrightarrow{OC}} = \frac{k+1}{k}$ or $k = \frac{1}{2.5}$ or using a ratio approach eg $(\overrightarrow{OC} : \overrightarrow{CD}) = k : 1 = 1 : 2.5$
			A1	cao

Q2

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB \rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a) M1: Attempts subtraction but may omit brackets
A1: cao (allow column vector notation)

(b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)
A1ft: $\lvert AB \rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Q3.

Question Number	Scheme	Marks
	$(15\mathbf{i}+\mathbf{j})+(5q\mathbf{i}-p\mathbf{j})+(-3p\mathbf{i}-q\mathbf{j})=\mathbf{0}$ $3p-5q=15$ $p+q=1$ $p=2.5 \quad q=-1.5$	M1 M1 A1 M1 A1 A1 **6**
	Notes	
	First M1 for equating the sum of the three forces to zero (can be implied by subsequent working) Second M1 for equating the sum of the **i** components to zero AND the sum of the **j** components to zero oe to produce TWO equations, each one being in p and q ONLY. First A1 for TWO correct equations (in any form) **N.B.** It is possible to obtain TWO equations by using $\lambda(3p-5q-15)=\mu(p+q-1)$ with TWO different pairs of values for λ and μ, with one pair not a multiple of the other e.g λ=1, μ=1 AND λ=1, μ=2. Third M1(independent) for attempt (either by substitution or elimination) to produce an equation in either p ONLY or q ONLY. Second A1 for p = 2.5 (any equivalent form, fractions do not need to be in lowest terms) Third A1 for q = - 1.5 (any equivalent form, fractions do not need to be in lowest terms)	

Q4.

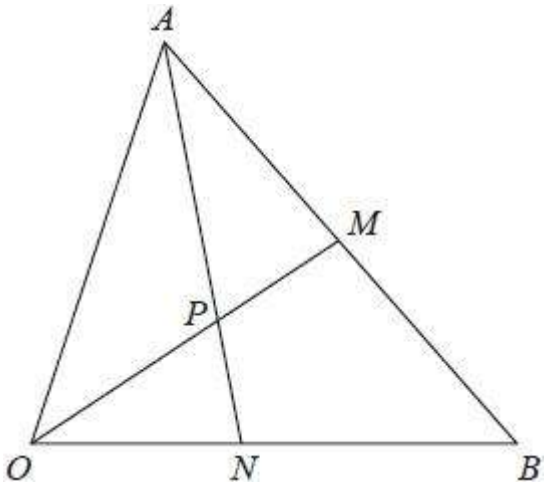
Question Number	Scheme	Marks
(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed $= \sqrt{1^2 + (-3)^2}$	M1
	$= \sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t = 2$ gives $\mathbf{v} = (-3\mathbf{i} + 3\mathbf{j})$	M1
	Bearing is 315°	A1
		(2)
(c)(i)	$1 - 2t = 0 \Rightarrow t = 0.5$	M1 A1
(ii)	$-(3t - 3) = -3(1 - 2t)$	M1 A1
	Solving for t	**DM**1
	t = 2/3, 0.67 or better	A1
		(6)
		[11]
	Notes for Question	
Q (a)	B1 for **i** – 3**j**. M1 for √ (sum of squares of cpt.s) A1 for √10, 3.2 or better	
Q (b)	M1 for clear attempt to sub $t = 2$ into given expression. A1 for 315.	
Q (c)	**(i)** First M1 for $1 - 2t = 0$. First A1 for $t = 0.5$. N.B. If they offer two solutions, by equating both the **i** and **j** components to zero, give M0. **(ii)** First M1 for $\frac{1-2t}{3t-3} = \pm(\frac{-1}{-3})$ o.e. (Must be an equation in t *only*) First A1 for a correct equation (the + sign) Second M1, dependent on first M1, for solving for t. Second A1 for 2/3, 0.67 or better.	

Gold Questions

Calculators may not be used

The total mark for this section is 29

Q1



OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$OP : PM = 3 : 2$

Work out the ratio $ON : NB$

(Total for Question 1 is 5 marks)

Q2

[*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by
$\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

(Total for Question 2 is 11 marks)

Q3

[*In this question* **I** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)

Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector 100 **j** m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\,\mathbf{i} + (2.7t - 54)\,\mathbf{j}]\text{metres}.$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

(Total for Question 3 is 13 marks)

Gold Mark Scheme

Q1

Answer	Mark	Mark scheme	Additional guidance
3 : 4	P1	starts process eg $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ oe	
	P1	for process to find $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}$ "$(\mathbf{b} - \mathbf{a})$"oe $(=\frac{1}{2}(\mathbf{a} + \mathbf{b}))$	
	P1	for process to find $\overrightarrow{AP} = -\mathbf{a} + \frac{3}{5}$ "$(\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b})$" oe or (indep) for $\overrightarrow{AN} = -\mathbf{a} +$ "k"$\mathbf{b}$	
	P1	process to find "k" using $\overrightarrow{AN} = -\mathbf{a} +$ "k"$\mathbf{b}$ as a multiple of $\overrightarrow{AP}$	
	A1	cao	
		ALTERNATIVE	
	P1	for producing OM to C such that AC is parallel to OB	Formal geometric reasoning relating to congruent and similar triangles is not required
	P1	for process to show that $MC = OM$, using congruent triangles ACM and BOM	
	P1	for process to find PC as a multiple of $OM/5$ $(= 7OM/5)$	
	P1	for process to find ON as a multiple of $AC(OB)$ $(= 3OB/7)$ using	
	A1	similar triangles ACP and NOP cao	

Q2

Question Number	Scheme	Marks
(a)	$\frac{(\mathbf{i}-4\mathbf{j})-(4\mathbf{i}-8\mathbf{j})}{0.5}; (\pm 6\mathbf{i} \pm 8\mathbf{j})$ $\sqrt{(\pm 6)^2+(\pm 8)^2}=10$	M1 A1 M1 A1 (4)
(b)	$\mathbf{r}=(4\mathbf{i}-8\mathbf{j})+t(-6\mathbf{i}+8\mathbf{j})$ $=(4\mathbf{i}-8\mathbf{j})-6t\mathbf{i}+8t\mathbf{j}$ $=(4-6t)\mathbf{i}+(8t-8)\mathbf{j}$ *	M1 A1 (2)
(c)	At 10 am, $\mathbf{r}=-2\mathbf{i}$ At 10.30 am, $\mathbf{r}=-5\mathbf{i}+4\mathbf{j}$ $\mathbf{l}=k\mathbf{i},\ k<-2$ $k=-5-4=-9$ $\mathbf{l}=-9\mathbf{i}$	M1 A1 A1 DM1 A1 (5) **11**

Q3

Question Number	Scheme	Marks
(a)	$\lvert\mathbf{v}\rvert = \sqrt{1.2^2 + (-0.9)^2} = 1.5 \text{ m s}^{-1}$	M1 A1 (2)
(b)	$(\mathbf{r}_H =) 100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j})$ m	M1 A1 (2)
(c)	$(\mathbf{r}_K =) 9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j})$ m	M1 A1
	$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}$ m **Printed Answer**	M1 A1 (4)
(d)	Meet when $\overrightarrow{HK} = \mathbf{0}$	
	$(9 - 0.45t) = 0$ and $(2.7t - 54) = 0$	M1 A1
	$t = 20$ from both equations	A1
	$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j})$ m	DM1 A1 cso
		(5)
		[13]

Platinum Questions

Calculators may not be used 

The total mark for this section is 16

1 Points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, respectively, relative to an origin O, and are such that OAB is a triangle with $OA = a$ and $OB = b$.

The point C, with position vector $\mathbf{c}$, lies on the line through O that bisects the angle AOB.

(a) Prove that the vector $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(4)

The point D, with position vector $\mathbf{d}$, lies on the line AB between A and B.

(b) Explain why $\mathbf{d}$ can be expressed in the form $\mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ for some scalar λ with $0 < \lambda < 1$

(2)

(c) Given that D is also on the line OC, find an expression for λ in terms of a and b only and hence show that

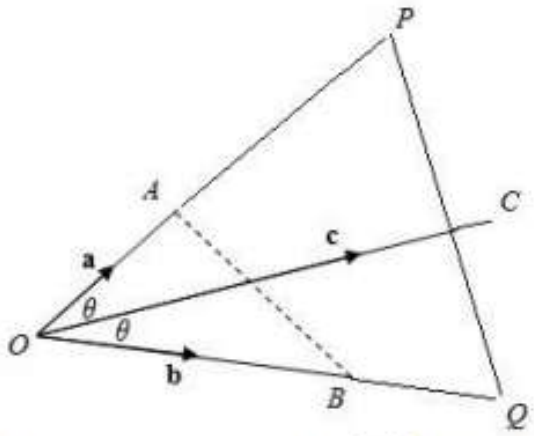
$$DA : DB = OA : OB$$

(8)

(+S2)

(Total for Question 1 is 16 marks)

Platinum Mark Scheme

Question	Scheme	Marks	Notes
5. (a)			(S+ for good diagram sketched)
	Let P and Q be points such that $\overrightarrow{OP} = b\mathbf{a}$ and $\overrightarrow{OQ} = a\mathbf{b}$.	M1	Extends OA and OB (may use unit vectors instead)
	Then $\lvert\overrightarrow{OP}\rvert = b\lvert\mathbf{a}\rvert = ba = ab = a\lvert\mathbf{b}\rvert = \lvert\overrightarrow{OQ}\rvert$ hence OPQ is isosceles.	A1	Deduce isosceles or equivalent.
	Hence the angle bisector from O is perpendicular to PQ.	M1	Use isosceles to deduce perpendicular
	But $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ} = b\mathbf{a} - a\mathbf{b}$ and hence as C is on the angle bisector, so $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.	A1	Draw correct conclusion.
		(4)	
(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \lambda\overrightarrow{AB} \Rightarrow \mathbf{d} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ $\Rightarrow \mathbf{d} = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ $(0 < \lambda < 1$ since D is between A and $B)$	M1 A1 (S+)	Sets up appropriate equation, either form. Correctly shown (Reasoning for λ)
		(2)	
(c)	($\overrightarrow{OD} = k\mathbf{c}$ and from (a) $\mathbf{c} = K \times \frac{1}{2}\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$ hence) $\overrightarrow{OD} = k'\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$	M1	Makes deduction that **d** is a multiple of **p**+**q**
	Hence $\mathbf{d} = k'\left(\overrightarrow{OP} + \overrightarrow{OQ}\right) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$	M1	Equates their **d** to **d** from (b)
	So $k'(b\mathbf{a} + a\mathbf{b}) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$	M1	Forms equation in **a** and **b**
	Therefore (since **a** and **b** are not parallel) $k'b = 1-\lambda$ and $k'a = \lambda$ $\Rightarrow \frac{\lambda}{a}b = 1-\lambda \Rightarrow \lambda = \frac{a}{a+b}$	M1 A1	Extracts simultaneous equations and solves for λ. (S+ for non-parallel reasoning)

Question	Scheme	Marks	Notes
	$\overrightarrow{AD} = (1-\lambda)\mathbf{a} + \lambda\mathbf{b} - \mathbf{a} = \lambda(\mathbf{b}-\mathbf{a}) \Rightarrow AD = \lambda\lvert\mathbf{b}-\mathbf{a}\rvert$ $\overrightarrow{BD} = (1-\lambda)\mathbf{a} + \lambda\mathbf{b} - \mathbf{b} = (1-\lambda)(\mathbf{a}-\mathbf{b}) \Rightarrow BD = (1-\lambda)\lvert\mathbf{b}-\mathbf{a}\rvert$ So $\frac{AD}{BD} = \frac{\lambda}{1-\lambda}$ $= \frac{a/a+b}{b/a+b} = \frac{a}{b} = \frac{OA}{OB}$	M1 dM1 A1	Correct work to establish ratio (may just be quoted) Give M0 if division of vectors is used. Substitutes in for λ **Given result** established
		(8)	
S2	S2 mark: Award S2 for a clear and concise solution that is EITHER - fully correct with no majorly incorrect vector notation used OR - that scores 12+ and includes at least 2 S+ points but may have some poor notation and be slightly laboured Award S1 for - a clear solution that scores 10+ marks with at least one S+ point.	**(2)**	
	Notes **(a)** S+ for a clearly labelled diagram drawn showing at least **a, b** and **c** **(b)** for the explanation of why $0 < \lambda < 1$ **(c)** S+ for reason given for being able to equate coefficients, e.g. vectors cannot be parallel since OAB is a triangle. S+ for any innovative ways used throughout the question.		
		Total 14 + 2 marks	